A Solution of Non-Linear Stochastic Delay Differential Equation on Asset Values for Capital Market Prices

[1]Amadi, I. U. [2]Tamunotonye, R., and [3]Azor P.A
[1] Department of Mathematics & Statistics, Captain Elechi Amadi Polytechnics, Port Harcourt, Nigeria, email: innocent.amadi@portharcourtpoly.edu.ng
[2] Department of Mathematics & Statistics, Ignatius Ajuru University of education, Rumuolumeni, Port Harcourt, Nigeria.
Nigeria, email: tonyesruth20@gmaill.com
[3]Department of Mathematics & Statistics, Federal University, Otuoke, Nigeria,
Email: azor.promise@gmail.com

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Abstract

The effect of time delay on the solution of Non-linear Stochastic Delay Differential Equation (SDDE) is considered. The problem were solved analytical by the method of Ito's to obtain a solution which gave birth to two additional solutions by the change of initial closing stock prices under multiplicative effects rate of returns. More so, various statistical tests were obtained effectively. To this end, the influences of the relevant parameters of stochastic quantities were all discussed in this paper.

Key words: *Stochastic Analysis Asset value, Multiplicative effects, and SDDE*

1.Introduction

Generally, investments are trading activities interrelated to risk which cannot be played down. Human beings actually take risk surviving; hence, risk play a significant role in the effective management of portfolio of investments since it aids the determination of changes in returns on the stock and portfolio, which provides for the investor a mathematical under pinning for investment decisions [1] Bonds, stocks, property, etc, are all examples of the risk interrelated to a security.

All the same, due to the risk engagement in portfolio of investments, the concept of insuring lives and properties, etc, is brought about by insurance companies. As a matter of fact, insuring companies share third party in their responsibility of financial outcomes or results. Risk transfer or risk sharing is the procedure where by insurance firm on financial out of its coverage duty in several ways with risk transfer agreement, risk among various insurance firms worldwide will be divided. Consequently, in the phenomenon of huge loses from financial situation as insurance firm will not encounter risk, specifically, reinsurance refers to division and distribution of risk. Basically, risk is a prevalent factor inasmuch as humans are concerned, since we secure risky or riskless assets appropriately. A better way of modelling these factors is as the trajectory or path of a diffusion process defined on several basic or fundamental probability space, having the geometric Brownian motion, the main too used as the established reference model [2] Modelling financial concepts cannot be hyperbolized because of its several applications in science and technology.

For example, [3] examined the maximization of the exponential utility and the minimization of the ruin probability and the results demonstrated the same or kind of investments scheme or approach for zero interest rate [4] Considered an optimal reinsurance and investment problem for insurer with jump diffusion risk process. [5] looked at the risk reserved for an insurer and a reinsurer to follow Brownian motion having drift and applied optimal probability of survival problem under proportional reinsurance and power utility preference. Similarly, [7] considered the excess loss of reinsurance and investment in a financial market and obtained optimal strategies. [8] engaged a problem of optimal reinsurance investment for an insurer having jump diffusion risk model when the asset price was control by a CEV model. [9] studied strategies of optimal reinsurance and investment for exponential utility maximization under different capital markets. [10] considered investment problem having multiple risky assets. [1] examined an optimal portfolio selection model for risky assets established on asymptotic power law behaviour where security prices follow a Weibull distribution. The research of [11] evaluated the stability of stochastic model of price fluctuation on the floor of the stock market, where exact steps were derived, which aided the determination of the equilibrium price and growth rate of stock shares. [12] studied the unstable property of stock market forces, making use of proposed differential equation model. [13] did a stochastic analysis of stock prices and their characteristics and obtained results which showed efficiency in the use of the proposed model for the prediction of stock prices. Similarly, [14] considered the stochastic formulations of some selected stocks in the Nigerian Stock Exchange (NSE), and the drift and volatility measures or quantities for the stochastic differential equations were obtained and the Euler-Maruyama technique for system of SDEs was applied in the stimulation of the stock prices. [8] produced the geometric Brownian motion and assessment of the correctness or exactness of the model, using detailed analysis of stimulated data. Furthermore, [15] considered stochastic problem of unstable stock market prices obtained conditions for determining the equilibrium price, required and adequate conditions for dynamic stability and convergence to equilibrium of the growth rate of the valued function of stocks. All the same, [16] looked at a stochastic problem of unstable prices at the floor of the stock market. From their evaluation, the equilibrium price and the market growth rate of shares were found out. Hence , so many scholars have written extensively on stock market prices such as [11-24] etc

Previous studies have therefore investigated similar problems but did not consider the effects of different initial stock prices and time delay in assessing asset values through multiplicative effects. In particular, some studies, for instance[19] ,[20] and [24] etc.

In this study we considered the effect of time delay on the solution of Non-linear Stochastic Delay Differential Equation (SDDE) . Apart from correctly posing the models for the assessment of asset values, we also solved in details by the method of Ito's to obtain a solution which gave birth to two additional solutions by the change of initial closing stock prices under multiplicative effects rate of returns. More so, various statistical tests were obtained to show that the asset values are physically consistent. To this end, this is the first study that has demonstrated the influences of different initial stock prices and time delay in respect to asset values in time varying investments.

The making ready of this paper is set as follows: Section 2.1 presents the preliminaries, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2.1: Preliminaries

At this stage we present few intricate definitions as stirring this dynamic area of study, hence we have as follows:

Definition 2.1.1: $\sigma-$ **algebra:** let Ω be a non-empty set and let F be a collection of subsets of Ω. Then F is said to be a $\sigma-$algebra if the properties are satisfied:
$i)$ $\phi \in \mathrm{F}$.
ii) given that a set $A \in \mathrm{F}$ then the compliment of A ie $A^c \in \mathrm{F}$.

iii) whenever the sequence $(A_i) \in \mathrm{F}$ for i=1,2,...,then $\bigcup_{n=1}\infty,\ A_n \in \mathrm{F}$.

Definition: 2.1.2Probability measure: let Ω be a non-empty set and let F be a $\sigma-$algebra of subsets of Ω**.** Then a function that assigns every set $\phi \in \mathrm{F}$ to a number in $[0,1]$ is called a probability measure P **.** its denoted as $\mathrm{P}(A)$ which is the probability of A such that we have the following:

i) $\mathrm{P}(\Omega)=1.$

ii) If $A_1, A_{2,} ...,$ is a sequence of disjoint sets in F then

$$\mathrm{P}\left(\cup_{n=1}^{\infty} A_n\right)=\sum_{n=1}^{\infty}\mathrm{P}(A_n)$$

The pair (Ω,F) is called a measurable space while the triple $(\Omega,\mathrm{F},\mathrm{P})$ is known a probability space [17].

Definition 2.1.3: **Filtration** : let Ω be a non-empty set and T be a fixed positive number and assume that for each $t\in[0,T]$ there is a $\sigma-$algebra $\mathrm{F}(t)$**.** Suppose $s \le t$ then every set $\mathrm{F}(s)$ is also in $\mathrm{F}(s)$ is also in $\mathrm{F}(t)$ is also in $\mathrm{F}(t)$**.** A filtration is a collection of $\sigma-$algebra $\mathrm{F}(t)$ for $0<t<T$.

Definition 2.1.3 : Normal Distribution: A normal distribution function is a peculiar distribution in probability theory and is usually used for modeling asset returns. A normal distribution is used in the Black-Scholes Partial differential equation to value European options. A normal distribution depends on two parameters.

(i) Mean, $\mu \in \square$, is the expectation of a random variable normal distribution.

(ii) Variance , $\sigma^2>0$, deals with the magnitude of the spread from the mean.

In Black-Scholes formula, normal distributions are used. The cumulative distribution, usually denoted as $\phi(\mathrm{X})$, is the probability that X will be equal to or less than X ,expressed as $F_x(x)=P(X\le x)$. A standard normal cumulative distribution function is defined as.

$$\phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{t^2}{2}}dt$$

A normal distribution is a symmetric distribution, which means that it touches around a vertical axis of symmetry. Obviously, there is a connection between any given points with same distance to the vertical axis. This relationship is defined in equation (5)

$$\phi(x)=1-\phi(-x)$$

Definition 3.6: Stochastic process: A stochastic process $X(t)$ is a relations of random variables $\{X_t(\gamma), t\in T, \gamma\in\Omega\}$, i.e, for each t in the index set T, $X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t. In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

It can also be seen as a statistical event that evolves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete.

Definition 3.7: A stochastic process whose finite dimensional probability distributions are all Gaussian.(Normal distribution).

Definition 3.9: A Stochastic Differential Equation is a differential equation with stochastic term. Therefore assume that $(\Omega, \mathrm{F}, \wp)$ is a probability space with filteration $\{f_t\}_t \geq 0$ and $W(t)=(W_1(t), W_2(t), \ldots, W_m(t))^T, t\geq 0$ an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g *as* follows

$$dX(t)=f(t, X(t))dt+g(t, X(t))dZ(t),\ 0\leq t\leq T, \tag{1}$$

$$X(0)=x_0, \tag{2}$$

where $T>0$, x_0 is an n-dimensional random variable and coefficient functions are in the form $f:[0,\mathrm{T}]\times\mathbb{R}^n$ *and* $g:[0,T]\times\mathbb{R}^n\rightarrow\times\mathbb{R}^{n\times n}$. SDE can also be written in the form of integral as follows:

$$X(t)=x_0+\int_0^t f(S, X(S))dS+\int_0^t g(S, X(S))dZ(S) \tag{3}$$

Where dX, dZ are terms in (1) known as stochastic differentials. The $\mathbb{R}^n$ is a valued stochastic process $X(t)$ satisfying (1).

Theorem 1.1: let $T>0$, be a given final time and assume that the coefficient functions $f:[0,\mathrm{T}]\times\mathbb{R}^n\rightarrow\mathbb{R}^n$ *and* $g:[0,T]\times\mathbb{R}^n\rightarrow\times\mathbb{R}^{n\times n}$ are continuous. Moreover, $\exists$ finite constant numbers λ *and* β such that $\forall t\in[0,\mathrm{T}]$ and for all $x, y\in\mathbb{R}^n$, the drift and diffusion term satisfly

$$\| f(t,x)-f(t,y)\|+\|\mathrm{g}(t,x)-g(t,y)\|\leq\lambda\| x-y\|, \tag{4}$$

$$\| f(t,x)\| + \| g(t,x)\| - g(t,x)\| \leq \beta(1+\| x\|). \tag{5}$$

Suppose also that x_0 is any $\mathbb{R}^n$ -valued random variable such that $E(\| x_0 \|^2) < \infty$. then the above SDE has a unique solution X in the interval $[0,T]$. Moreover, it satisfies $E\left(\sup_{0\leq t\leq T} \| X(t)\|^2\right) < \infty$. the proof of the theorem 1.1 is seen in [18].

Theorem 3.2:(Ito's lemma). Let $f(S,t)$ be a twice continuous differential function on $[0,\infty)\times A$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0 \ ,$$

Applying Taylor series expansion of F gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}(dS_t)^2 + \text{higer order terms}\ (h.\text{o},\text{t})\ ,$$

So, ignoring h.o.t and substituting for dS_t we obtain

$$dF_t = \frac{\partial F}{\partial S_t}(a_t dt + bdz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}(a_t dt + bdz(t))^2 \tag{6}$$

$$= \frac{\partial F}{\partial S_t}(a_t dt + bdz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} b_t^2 dt, \tag{7}$$

$$= \left(\frac{\partial F}{\partial S_t} a_t + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} b_t^2\right) dt + \frac{\partial F}{\partial S_t} b_t dz(t) \tag{8}$$

More so, given the variable $S(\text{t})$ denotes stock price, then following GBM implies and hence, the function $F(S,t)$,Ito's lemma gives:

$$dF = \left(\mu S\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 F}{\partial S^2}\right) dt + \sigma S\frac{\partial F}{\partial S} dz(t) \tag{9}$$

3.1 .1 Mathematical Formulation of the Problem

A non-linear Stochastic Delay Differential Equation (SDDE) with multiplicative effects for rate of return of investments is considered. The influence of time delay on the value of asset is also considered consistently through multiplicative effects. The stock volatility is constant throughout the investment periods. Various impacts on initial stock prices were reflected through the value of asset over time; which results to test of normality. The dynamics of asset value and its return rates is said to have a complete probability space $(\Omega, \text{F}, \wp)$ such that a finite time investment horizon $T > 0$.[23]. Hence , we have the equation governing the process as:

$$dX_i\left(t\right)=\left(t-\tau\right)X_i(t)dt+\sigma X_1(t)dZ(t) \tag{10}$$

where μ is an expected rate of returns on stock, σ is the volatility of the stock , dt is the relative change in the price during the period of time and Z is a Wiener process .Following the method of [19-20] on rate of returns gives as:

$$\begin{gathered} R_t := \left(\lambda_1\lambda_2\right)^2,... \\ \text{where } t=1,2... \end{gathered} \tag{11}$$

Using (10) and (11) gives the following system of delay stochastic differential equations :

$$dX_1(t)=\left(t-\tau\right)\left(\theta_1\theta_2\right)^2 \mathrm{X}_1(t)dt+\sigma X_1(t)dZ^{(1)}(\mathrm{t}) \tag{12}$$

Where $\mathrm{X}_1(\mathrm{t}), \mathrm{X}_2(\mathrm{t})$ and $S_3(\mathrm{t})$. are underlying stocks with the following initial conditions:

$$\mathrm{X}_1(0)=X_0\,,\, t>0 \tag{13}$$

$$X_2\left(0\right)=e^t \;\;, t>0 \tag{14}$$

$$X_5\left(0\right)=e^{-t} \;\;, t>0$$

The expression dZ , which contains the randomness that is certainly a characteristic of asset prices is called a Wiener process or Brownian motion. λ_1 *and* λ_2 represents rate of returns of first and second investments respectively, $\lambda_1\lambda_2$ is multiplicative effects.

3.1.2 Method of Solution

The model (12) is stochastic delay differential equations and initial conditions whose solutions are not trivial. We implement the methods of Ito's lemma in solving for $X_1(t), X_2(t)$ *and* $X_5(t)$ To grab this problem we note that we can forecast the future worth of the asset with sureness.

From (12) Let $f\left(X_1,t\right)=\ln X_1$ so differentiating partially gives

$$\frac{\partial f}{\partial X_1}=\frac{1}{X_1},\;\frac{\partial^2 f}{\partial X_1^2}=-\frac{-1}{X_1^2},\;\frac{\partial f}{\partial t}=0 \tag{16}$$

According to Ito's gives

$$df\left(X_1,t\right)=\sigma X_1\frac{\partial f}{\partial X_1}dZ(t)+\left((\theta_1\theta_2)^2X_1(t)\frac{\partial f}{\partial X_1}+\frac{1}{2}\sigma^2X_1^2\frac{\partial^2 f}{\partial X_1^2}+\frac{\partial f}{\partial t}\right)dt \tag{17}$$

Substituting (12) and (16) into (17) gives

$$= \sigma \cancel{X_1} \frac{1}{\cancel{X_1}} dZ(t) + \left(\left(t-\tau\right)(\theta_1\theta_2)^2 \cancel{X_1}(t) \frac{1}{\cancel{X_1}} + \frac{1}{2}\sigma^2 \cancel{X_1}^2 \left(-\frac{1}{\cancel{X_1}^2}\right) + 0 \right) dt$$

$$= \sigma dZ(t) + \left(\left(t-\tau\right)(\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2 \right) dt$$

Integrating both sides , talking upper and lower limits gives

$$\int_0^t d\ln X_1 = \int_0^t df\left(X_u, u\right) = \left(t-\tau\right)\int \left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2 \right) du + \int_0^t \sigma dZ(t)$$

$$\ln X_1 - \ln X_0 = \left(t-\tau\right)\left((\theta_1\theta_2)^2 u - \frac{1}{2}\sigma^2 u \right)\Big|_0^t + \left(\sigma Z u\right)\Big|_0^t \qquad (18)$$

$$\ln\left(\frac{X_1}{X_0}\right) = \left(t-\tau\right)\left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2 \right) t + \sigma Z(t)$$

Taking the ln of the both sides

$$X_1(t) = X_0 \exp\left(t-\tau\right)\left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2 \right) t + \sigma Z(t) \qquad (19)$$

Applying other initial stock in (14) and (15) yields

$$X^{*}{}_2(t) = e^t \, \mathrm{e}\left(t-\tau\right)\left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2 \right) t + \sigma Z(t) \qquad (20)$$

$$X^{*****}{}_5(t) = \frac{1}{e^t} \mathrm{e}\left(t-\tau\right)\left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2 \right) t + \sigma Z(t) \qquad (21)$$

Table 1: The impact of time delay in the assessment of multiplicative Asset values through the solution below: $X_1\left(t\right) = X_0 \exp\left\{ \left(t-\tau\right)\left(\left(\theta_1\theta_2\right)^2 - \frac{1}{2}\sigma^2 \right) t + \sigma dz\left(t\right) \right\}$ $\quad t = 2, \quad dz = 1$

τ	X_0	σ	$\left(\theta_1\theta_2\right)$	$X_1\left(t\right)$	$\left(\theta_1\theta_2\right)$	$X_1\left(t\right)$
0.1		0.2	1.0000	253.0123	2.0000	22599490.33
0.2	5.00	0.2	1.0000	207.9792	2.0000	10195305.35
0.3		0.2	1.0000	170.9614	2.0000	4599406.873
0.4		0.2	1.0000	146.1537	2.0000	2157927.063
0.5	5.20	0.2	1.0000	120.1401	2.0000	973505.3755
0.6		0.2	1.0000	98.7566	2.0000	439177.3626
0.7		0.2	1.0000	57.7621	2.0000	140974.291
0.8	3.70	0.2	1.0000	47.4812	2.0000	63597.7149
0.9		0.2	1.0000	39.0301	2.0000	28690.8295
1.0		0.2	1.0000	28.7015	2.0000	11578.9977

1.1	3.31	0.2	1.0000	23.5930	2.0000	5223.6318
1.2		0.2	1.0000	19.3937	2.0000	2356.5364

The multiplicative effects $(\lambda_1\lambda_2)$ as it labels across Tables 1, 2 and 3 shows the levels of stock returns and its value of asset which is assume to follows multiplicatively through the impact of delay. See columns 4 and 6 of Tables. Therefore, when stock return rates are fixed from 1.0000-2.0000 respectively it duplicates the variations in the value of assets see columns 5 and 7 respectively of the Tables. This asset value as seen above informs an investor the common changes of investments, as well as changes in circumstances that may affect an investment over time. This remark enhances viable investment decisions so as to maximize greater profits.

On the contrary, in the same Tables 1 , 2 and 3 show a little increase in time delay suggestively reduces asset values through multiplicative effects. This is correct, because as time passes some certain asset values continue to devalues over time; example of such assets are cars, chairs and machines etc. This remark is an guarding rule to an investor on how to effectively cope such depreciating assets during the trading days.

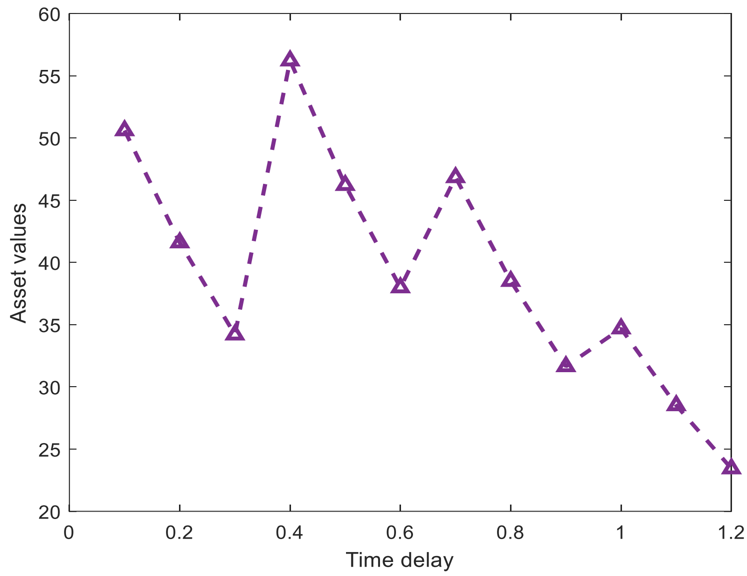

Figure 1 : Profile of Asset values with variations of Time Delay when initial price (X_0)

It can be noticed in Figure 1 the pictorial trend and impact of time delay to precisely measure asset value in time varying investments which is financially indexed in millions of naira. This is obviously informative to an investor whose aim and passion is to maximize profit hence it specifies price changes on the trend lines throughout the trading days. The price changes can be attributed to a lot of influences in the society over time.

Table 2: The impact of time delay in the assessment of multiplicative Asset values through the solution below: $X_1(t) = e^t \exp\left\{(t-\tau)\left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2\right)t + \sigma dz(t)\right\}$

$dz = 1, t = 2$ and $X_0 = e^t$

τ	S_0	σ	$(\theta_1\theta_2)$	$X_1(t)$	$(\theta_1\theta_2)$	$X_1(t)$
0.1	7.3891	0.2	1.0000	373.9066	2.0000	33397978.81
0.2	7.3891	0.2	1.0000	307.3558	2.0000	15066826.15
0.3	7.3891	0.2	1.0000	252.6502	2.0000	6797095.465
0.4	7.3891	0.2	1.0000	207.6816	2.0000	3066372.857
0.5	7.3891	0.2	1.0000	170.7168	2.0000	1383332.417
0.6	7.3891	0.2	1.0000	140.3313	2.0000	624062.5866
0.7	7.3891	0.2	1.0000	115.3540	2.0000	281533.2794
0.8	7.3891	0.2	1.0000	94.8224	2.0000	127008.0744
0.9	7.3891	0.2	1.0000	77.9452	2.0000	57297.1373
1.0	7.3891	0.2	1.0000	64.0719	2.0000	25848.4506
1.1	7.3891	0.2	1.0000	52.6679	2.0000	11661.0084
1.2	7.3891	0.2	1.0000	43.2937	2.0000	5260.6293

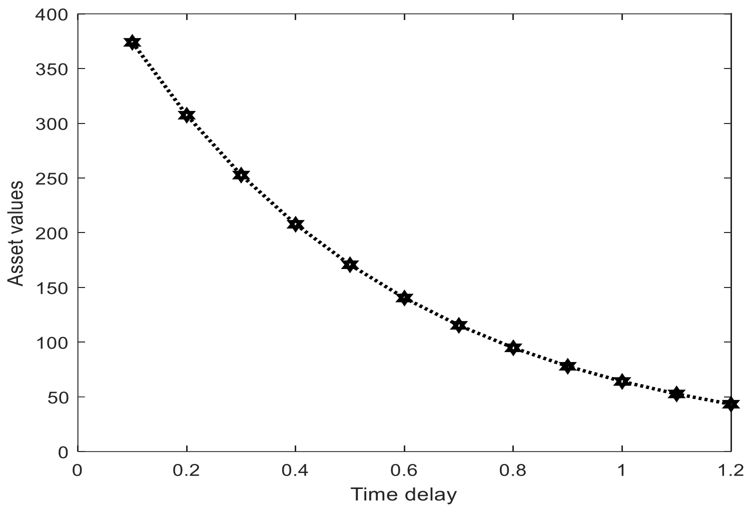

Figure 2 : Profile of Asset values with variations of Time Delay when the initial stock is exponential through multiplicative effects.

Clearly it can be observed in Figures 2 and 3 indicate exponential growth shape that is dominantly, incremental as time passes. Having such exponential growth in the investment means the trading business is greatly profiting in millions of naira over life of the assets. This type of situation can interest investor to be focus in the rate of turnover as the businesses continue to grow exponentially.

Table 3: The impact of time delay in the assessment of multiplicative Asset values through the solution below:

$$X_5(t) = \frac{1}{e^t}\exp\left\{(t-\tau)\left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2\right)t + \sigma dz(t)\right\}$$

where $dz = 1, t = 2$ and $S_0 = \frac{1}{e^t}$

τ	S_0	σ	$(\theta_1\theta_2)$	$X_1(t)$	$(\theta_1\theta_2)$	$X_1(t)$
0.1	0.1353	0.2	1.0000	6.8465	2.0000	611542.2085
0.2	0.1353	0.2	1.0000	5.6279	2.0000	275884.9628
0.3	0.1353	0.2	1.0000	4.6262	2.0000	124459.95
0.4	0.1353	0.2	1.0000	3.8028	2.0000	56147.6022
0.5	0.1353	0.2	1.0000	3.1260	2.0000	25329.8610
0.6	0.1353	0.2	1.0000	2.5695	2.0000	11427.0572
0.7	0.1353	0.2	1.0000	2.1122	2.0000	5155.0869
0.8	0.1353	0.2	1.0000	1.7363	2.0000	2325.6137
0.9	0.1353	0.2	1.0000	1.4272	2.0000	1049.1539
1.0	0.1353	0.2	1.0000	1.1732	2.0000	473.3047
1.1	0.1353	0.2	1.0000	0.9644	2.0000	213.5219
1.2	0.1353	0.2	1.0000	0.7927	2.0000	96.3261

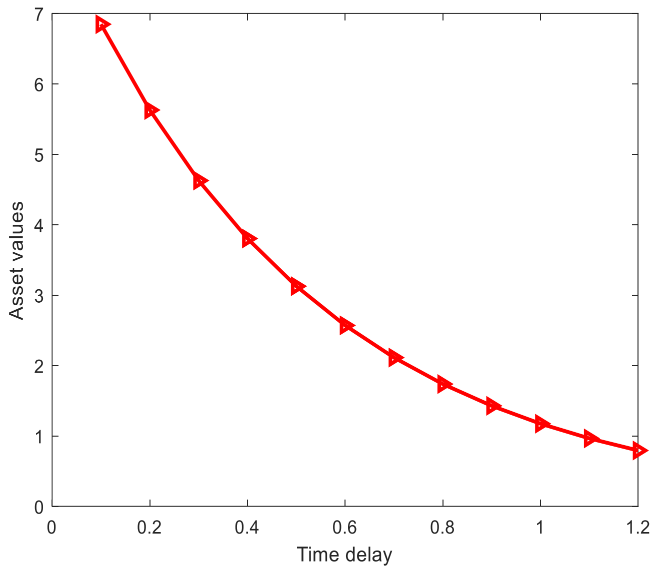

Figure 3 : Profile of Asset values with variations of Time Delay when the initial stock is inverse exponential through multiplicative effects.

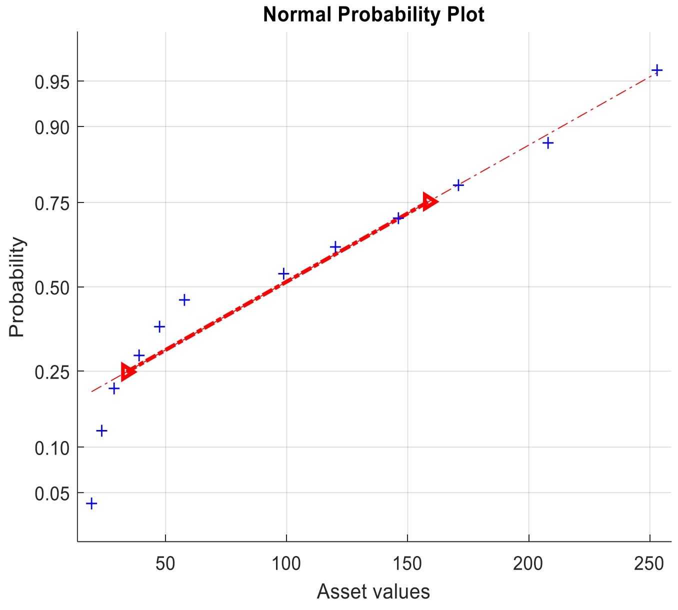

Figure 4 : Normal probability plot on Asset values when the initial stock price is (X_0)

The graphical description of Figure 4 shows that the two asset values through multiplicative effects of Table 1 come from the identical distribution. They are statistically significant and correlated. The plot portrays a significant sure event which is highly beneficial to investor over in a time varying investments and with this scenario; profit margin is unavoidable and decisions can be properly taken to enrich decent management of the business.

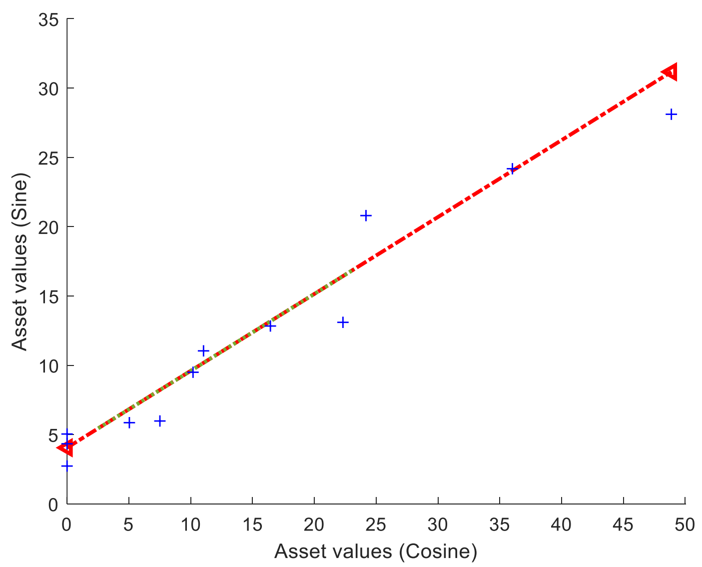

Figure 5 : Quantile-Quantile (QQ) plot assessment on two different Asset values when the initial stock prices is both exponential and exponential inverse

It can be seen in Figure 5 that the QQ plot indicates that the two exponential and inverse exponential asset values with multiplicative effects of Tables 2 and 3 comes from a common distributions. They are statistically significant, correlated and have lots of financial remunerations hence it trades around the normal distribution.

4.1 Conclusion

This study, considered stochastic analysis of discrepancies of initial stock prices through multiplicative effects with the impact of time delay parameter in the model. The asset values were obtained which all follows exponential trend of business in time varying investments. The normality probability and QQ plots all were statistically significant to enhance the issues in financial market for an assessment of asset values for capital market investments.

However, the present paper considered SDDE with multiplicative effects to realistically assess asset values, future study should incorporate control measures for the assessment of asset values.

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